TD 2

$f : \mathbb{R}^n \to \mathbb{R}$ $\mathcal{C}^1$; $u_1 : \mathbb{R} \to \mathbb{R}$ $\mathcal{C}^1$; ... ; $u_n : \mathbb{R} \to \mathbb{R}$

$x \mapsto f(x)$; $t \mapsto u_1(t)$; $t \mapsto u_n t$

$g : \mathbb{R} \to \mathbb{R}$

$t \mapsto g(t) = f(u_1(t), u_2(t) \dots u_n(t))$.

$g'(t) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(u_1(t), u_2(t), \dots, u_n(t)) u_i'(t)$

exercice 1

$f : \mathbb{R}^3 \to \mathbb{R}$

$g : \mathbb{R} \to \mathbb{R}$.

$(x, y, z) \mapsto f(x, y, z)$

$g(t) = t^2 f(3t+2, t^2, e^t)$

On a $u_1(t) = 3t+2$ $u_2(t) = t^2$ et $u_3(t) = e^t$.

On a $g'(t) = \frac{dg}{dt}(t) = 2t \times f(3t+2, t^2, e^t) + t^2 \times \frac{d}{dt}[f(3t+2, t^2, e^t)]$

$= 2t \times f(3t+2, t^2, e^t) + t^2 \left[\frac{\partial f}{\partial x}(3t+2, t^2, e^t) 3 + \frac{\partial f}{\partial y}(3t+2, t^2, e^t) 2t + \frac{\partial f}{\partial z}(3t+2, t^2, e^t) e^t\right]$.

$= 2t \times f(3t+2, t^2, e^t) + 3t^2 \frac{\partial f}{\partial x}(3t+2, t^2, e^t) + 2t^3 \frac{\partial f(-)}{\partial y} + e^t t^2 \frac{\partial f(-)}{\partial z}$

(Suite après).

exercice 1 (suite)

2) $g'(t) = 2t f(\_) + 3t^2 \frac{\partial f}{\partial x}(\_) + 2t^3 \frac{\partial f}{\partial y}(\_) + t^2 e^t \frac{\partial f}{\partial z}(\_)$

$g''(t) = \frac{d}{dt}[g'(t)] = \frac{d}{dt}[\ \ ]$

$= 2 \times f(\_) + 2t \times \frac{d}{dt}[f(\_)] + 6t \frac{\partial f}{\partial x}(\_) + 3t^2 \frac{d}{dt}$

$[\frac{\partial f}{\partial x}(\ \ )] + 6t^2 \frac{\partial f}{\partial y}(\_) + 2t^3 \times \frac{d}{dt}[\frac{\partial f}{\partial y}(\_)] +$

$2te^t \times \frac{\partial f}{\partial z}(\_) + t^2 e^t \times \frac{d}{dt}[\frac{\partial f}{\partial z}(\_)]$

$\frac{d}{dt}[\frac{\partial f}{\partial x}(3t+2, t^2, e^t)]$

$= \frac{\partial^2 f}{\partial x^2}(3t+2, t^2, e^t) \times 3 + \frac{\partial^2 f}{\partial y \partial x}(\_) 2t + \frac{\partial^2 f}{\partial z \partial x}(\_) e^t$

$\frac{d}{dt}[\frac{\partial f}{\partial y}(3t+2, t^2, e^t)]$

$= \frac{\partial^2 f}{\partial x \partial y}(\_) 3 + \frac{\partial^2 f}{\partial^2 y}(\_) 2t + \frac{\partial^2 f}{\partial z \partial y}(\_) e^t$

$\frac{d}{dt}[\frac{\partial f}{\partial z}(3t+2, t^2, e^t)]$

$= \frac{\partial^2 f}{\partial x \partial z}(\_) 3 + \frac{\partial^2 f}{\partial y \partial z}(\_) 2t + \frac{\partial^2 f}{\partial^2 z}(\_) \times e^t$

Rappel

$f : \mathbb{R}^n \to \mathbb{R}$, $x \mapsto f(x)$ $\quad u_1 : \mathbb{R}^p \to \mathbb{R}$, $(s_1, \dots, s_p) \mapsto u_1(s_1, \dots, s_p)$ $\quad \dots \quad u_m : \mathbb{R}^p \to \mathbb{R}$, $s \mapsto u_m(s)$

$g : \mathbb{R}^p \to \mathbb{R}$, $s \mapsto g(s) = f(u_1(s), \dots, u_m(s))$ $\quad x = (x_1, x_2, \dots, x_m)$, $s = (s_1, \dots, s_p)$

$i \in [\![1, p]\!]$ $\quad \frac{\partial g}{\partial s_i}(s) = \frac{\partial}{\partial s_i}[f(u_1(s), \dots, u_m(s))] =$

$\forall i \in [\![1, p]\!]$ $\quad \frac{\partial g}{\partial s_i}(s) = \sum_{j=1}^{m} \frac{\partial f}{\partial x_j}(u_1(s), u_2(s), \dots, u_p(s)) \frac{\partial u_j}{\partial s_i}(s)$.

Exercice 2

$f : \mathbb{R}^3 \to \mathbb{R}$ tq $(x, y, z) \to f(x, y, z)$.

$g : \mathbb{R}^2 \to \mathbb{R}$ tq $g(s, t) = (2s + t) f(s^2 + t, 2t, 3s + t)$

1) $\frac{\partial g}{\partial s}(s, t) = 2 f(s^2 + t, 2t, 3s + 2t) + (2s + t) \frac{\partial}{\partial s}[f(s^2 + t, 2t, 3s + 2t)]$

$= 2 f(s^2 + t, 2t, 3s + 2t) + (2s + t)\left[\frac{\partial f}{\partial x} 2s + \frac{\partial f(-)}{\partial y}(0) + 3\frac{\partial f(-)}{\partial z}\right]$

$= 2 f(s^2 + t, 2t, 3s + 2t) + 2s(2s + t)\frac{\partial f(-)}{\partial x} + 3(2s + t)\frac{\partial f(-)}{\partial z}$.

$\frac{\partial g}{\partial t}(s, t) = f(s^2 + t, 2t, 3s + t) + (2s + t)\left[\frac{\partial f}{\partial x} \times 1, 2\frac{\partial f}{\partial y}, 1\frac{\partial f}{\partial z}\right]$.

$= f(s^2 + t, 2t, 3s + 2t) + (2s + t) \times \frac{\partial f(-)}{\partial x} + 2(2s + t)\frac{\partial f}{\partial y} + (2s + t)\frac{\partial f}{\partial z}$

~~$g'(t) = 2f(-) + 2tx$~~

exercício 2 (suite)

$$\frac{\partial g}{\partial s}(s,t) = 2f(\_) + 2s(2s+t)\frac{\partial f}{\partial x}(\_) + 3(2s+t)\frac{\partial f}{\partial z}(\_)$$

$$\frac{\partial^2 g}{\partial^2 s} = \frac{\partial}{\partial s}\left[g\frac{\partial g}{\partial s}(s,t)\right] = \frac{\partial g}{\partial s}[\quad]$$

$$= 0 + 2\times\frac{\partial}{\partial s}[f(\_)] + (8s+2t)\frac{\partial f}{\partial x}(\_) + 2s(2s+t)$$

$$\frac{\partial}{\partial s}\left[\frac{\partial f}{\partial x}(\_)\right] + 6\frac{\partial f}{\partial z}(\_) + 3(2s+t)\frac{\partial g}{\partial s}\left[\frac{\partial f}{\partial z}(\_)\right]$$

$$\frac{\partial}{\partial s}\left[\frac{\partial f}{\partial x}[s^2+t, 2t, 3s+2t]\right]$$

$$\frac{\partial^2 f}{\partial x \partial y}(\_)2s + \frac{\partial^2 g}{\partial y \partial x}(\_)0 + \frac{\partial^2 f}{\partial z \partial x}(\_)3$$

$$\frac{\partial}{\partial s}\left[\frac{\partial f}{\partial z}(s^2+t, 2t, 3s+2t)\right]$$

$$\frac{\partial^2 f}{\partial x \partial z}(\_)2s + \frac{\partial^2 g}{\partial^2 z}(\_)3$$

exercice 3

$f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ avec $(x,y,3) \longmapsto f(x;y;3)$

$g : \mathbb{R} \longrightarrow \mathbb{R}$ tq $g(x) = x^2 f(3x+2, x^2, e^x)$

on a $g'(x) = \frac{d}{dx}(x) = 2x \times f(-) + x^2 \times \frac{\partial}{\partial x}[f(-)]$

$= 2x \times f(-) + x^2 \left[\frac{\partial f}{\partial x}(-)\, 3,\ 2x \frac{\partial f}{\partial y}(-),\ \frac{\partial f}{\partial 3}(-) e^x \right]$

$= 2x f(-) + 3x^2 \frac{\partial f}{\partial x}(-) + 2x^3 \frac{\partial f}{\partial y}(-) + x^2 e^x \frac{\partial f}{\partial 3}(-)$.

exercice 4

$f : \mathbb{R}^3 \to \mathbb{R}$ tq $(x;y;3) \longmapsto f(x;y;3)$

$g : \mathbb{R}^2 \to \mathbb{R}$ " $g(x,y) = (2x+4) f(x^2+4, 24, 3x+24)$

$\frac{\partial g}{\partial x}(x,4) = 2 \times f(-) + (2x+4) \times \frac{\partial}{\partial x}[f(-)]$

$= 2 \times f(-) + (2x+4) \times \left(\frac{\partial f}{\partial x}(-) 2x + \frac{\partial f}{\partial 4}(-) 0 + \frac{\partial f}{\partial 3}(-) 3 \right)$

$= 2 \times f(-) + 2x(2x+4) \times \frac{\partial f}{\partial x} + 3 \times (2x+4) \times \frac{\partial f}{\partial 3}$.

$$\frac{\partial g}{\partial y}(x,y) = 1 \times f(-) + (2x+y) \times \frac{\partial}{\partial x}[f(-)]$$

$$= f(-) + (2x+y) \times \left(\frac{\partial f}{\partial x}(-)1 + \frac{\partial f}{\partial y}(-)2 + \frac{\partial f}{\partial z}(-)2 \right)$$

$$= f(-) + (2x+y) \times \frac{\partial f}{\partial x}(-) + 2 \times (2x+y) \times \frac{\partial f}{\partial y} + 2(2x+y) \times \frac{\partial f}{\partial z}$$

exercice 5

$f : \mathbb{R}^3 \to \mathbb{R}$ tq $f(x;y;z) = x + 2yz^3$

1) $\frac{\partial f}{\partial x}(x;y;z) = 1$ $\quad \frac{\partial f}{\partial y}(x;y;z) = 2z^3$

$\frac{\partial f}{\partial z}(x;y;z) = 6yz^2$

2) $g : \mathbb{R}^2 \to \mathbb{R}$ tq $g(x;y) = f(x;y;z)$

a) $g(x;y) = x + 2yx^3$

b) $\frac{\partial g}{\partial x}(x;y) = 1 + 6yx^2$ $\quad \frac{\partial g}{\partial y}(x;y) = 2x^3$

c) $\left.\begin{array}{l} \frac{\partial f}{\partial x}(x;y;x) = 1 \\ \frac{\partial f}{\partial x}(x;y) = 1 + 6yx^2 \end{array}\right\}$ c'est différent